Lecture Notes: Section Modulus Calculations

Recall for hull girder bending: $\sigma_b(y) = \frac{My}{I}$ FOS $= \frac{\sigma_Y}{\sigma_b}$

Section modulus: $Z_{deck} = \frac{I}{y_{deck}}$ and $Z_{keel} = \frac{I}{y_{keel}}$ thus $\sigma_{deck} = \frac{M}{Z_{deck}}$ and $\sigma_{keel} = \frac{M}{Z_{keel}}$

Units: in^3 (sometimes $in^2 ft$)

Bending moment at each section is calculated by integration of the load distribution (along the length of the ship)

So, a naval architect needs to calculate Z_{deck} and Z_{keel} to make sure a proposed design (structural scantlings) is acceptable, or if a damaged ship is structurally sound.

Approach:

Parallel Axis Theorem for Moment of Inertia (2nd moment of area)

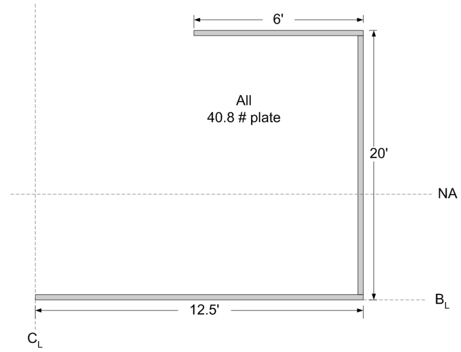
$$I_x = I_0 + A \cdot h^2$$

I_x = moment of inertia (2nd moment of area) of area about some axis of interest (x)
I_0 = moment of inertial (2nd moment of area) of area through it's centroid
A = cross sectional area
h = distance from centroid of area to axis of interest

Steps for calculating section modulus:
1. Calculate the area of each component (a)
2. Calculate the height of each component area (centroid) above the baseline (h)
3. Calculate the 1st moment of each component area about the baseline (ah)
4. Calculate the moment of inertia (2nd moment of area) of each component about the baseline (ah^2)
5. Calculate the moment of inertia of each component about its own horizontal centroidal axis (i)

   For vertical or horizontal plate/shape (breadth b, depth d): $i = \frac{bd^3}{12}$
6. Calculate the height of the NA above the baseline ($h_{NA} = \sum ah / \sum a$)
7. Calculate the moment of inertia of the total section about the baseline ($I_{BL} = \sum ah^2 + \sum i$)
8. Calculate the moment of inertia of the total section about the NA ($I_{NA} = I_{BL} - Ah_{NA}^2$)
9. Calculate the section modulus for deck and keel

Example: Box-shaped barge from lecture 4



Create a table (see also Hughes fig. 3.15 and table 3.5)

Item	Scantlings (b x d) (in)	Area a (in^2)	Height ABL h (in)	1st moment ah (in^3)	2nd moment about BL ah^2 (in^4)	2nd moment about own centroid i (in^4)
Deck	72 x 1	72	239.5	17,208	4,129,938	6
Side pl	1 x 238	238	120	28,560	3,427,200	1,123,439
Bottom pl	150 x 1	150	0.5	75	37.5	12.5
Total (½ section)						∑i = 1,123,457.5

Height of NA above BL: $h_{NA} = \dfrac{\sum ah}{\sum a} = \dfrac{45{,}843}{460} = 99.66\, in \approx 100\, in \approx 8.3\ ft$

Total Moment of inertia of section about NA:

$$I_{BL} = I_{NA} + A \cdot h_{NA}^{\ 2} \rightarrow$$

$$I_{NA} = I_{BL} - A \cdot h_{NA}^{\ 2} = \underbrace{\sum i + \sum ah^2}_{I_{BL}} - \left(\sum a\right) \cdot h_{NA}^{\ 2} = 1{,}123{,}457.5 + 7{,}557{,}175.5 - (460)\cdot(99.66)^2 = 4{,}111{,}860\, in^4$$

…but this is only ½ the section, so $\boxed{I = 8{,}223{,}720 \text{ in}^4}$

Max stress and FOS ?

$y_{max} = y_{deck} = 140$ in so $Z_{deck} = \dfrac{I}{y_{deck}} = \dfrac{8{,}223{,}720 \text{ in}^4}{140 \text{ in}} = 58{,}741 \text{ in}^3$

From the loading example: M_{max} = 34,800 ft-LT

$$\sigma_{deck} = \frac{M}{Z_{deck}} = \frac{34{,}800 \text{ ft - LT}}{58{,}741 \text{ in}^3}\left(12\frac{in}{ft}\right)\left(2240\frac{lb}{LT}\right) = 15{,}925\frac{lb}{in^2}$$

$$\text{FOS} = \frac{\sigma_Y}{\sigma_b} = \frac{36\, ksi}{15.9\, ksi} \approx 2.3$$

Notes:

This is still water! Wave w/ trough amidships (sagging) worse! (fix by distributing load)

This is a compressive stress, so the deck plating could also fail due to buckling …later

Only "longitudinally-continuous" and "rigidly-mounted" structure counts (not all structure is effective)

General rule of thumb: 40% L_{BP}

"Shadow Zones" near hatch openings & discontinuities

For inclined plate (width w, thickness t, angle to horizontal θ): $i = \dfrac{wt(w^2 \sin^2\theta + t^2 \cos^2\theta)}{12}$

For curved plate (radius r, area a): $i = \left(\dfrac{1}{2} - \dfrac{4}{\pi^2}\right) ar^2 \quad h = \dfrac{(\pi - 2)}{\pi} r$